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A model for self-tuning the cosmological constant

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Abstract

The vanishing cosmological constant in the four dimensional space-time is obtained in a 5D Randall-Sundrum model with a brane (B1) located at $y = 0$. The matter fields can be located at the brane. For settling any vacuum energy generated at the brane to zero, we need a three index antisymmetric tensor field A_{MNP} with a specific form for the Lagrangian. For the self-tuning mechanism, the bulk cosmological constant should be negative.

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The cosmological constant problem is the most serious hierarchy problem known for more than two decades [1], and there appeared several attempts [2] to understand the hierarchy between the cosmological constant Λ and the Planck mass $M_P \equiv 2.44 \times 10^{18}$ GeV. This problem has become even more difficult with the recent observation of the small but non-vanishing vacuum energy of order $(0.003\ \mathrm{eV})^4$ [3]. The quintessences have been considered to explain the smallness of this tiny vacuum energy [4], but the bottom line of these ideas is that there exists a solution of the cosmological constant problem.

The difficulty of solving the cosmological constant problem in the four dimensional space time lies in that the limit $\Lambda \to 0$ does not introduce any new symmetry. Thus, it may be necessary to go beyond the 4D space-time or introduce a more general form of the Lagrangian. In this Letter, we consider a solution of the cosmological constant problem with one extra dimension. In particular, we work with one brane located at $y = 0$ (B1 brane) where y is the 5th dimension, which is the so-called Randall-Sundrum II model [5].

The action can be written as

$$S = \int d^4x \int dy \sqrt{-g} \left(\frac{1}{2} R + \frac{2 \cdot 4!}{H_{MNPQ} H^{MNPQ}} - \Lambda_b + \mathcal{L}_m \delta(y) \right) \tag{1}$$

where we put the brane B1 at $y = 0$ and the brane tension at B1 is $\Lambda_1 \equiv - < \mathcal{L}_m >$. We set the fundamental mass parameter M as 1 and we will recover the mass M wherever it is explicitly needed. We assume a Z_2 symmetry of the solution, $\beta(-y) = \beta(y)$. We introduced the three index antisymmetric tensor field A_{MNP} whose field strength is denoted as H_{MNPQ}. The action contains the $1/H^2$ term which does not make sense if H^2 does not develop a vacuum expectation value. We anticipate that this term constitutes a part of the gravitational interactions, and hence the renormalizability is not considered in this paper. If there results a good solution for the cosmological constant problem, it can be more seriously considered as a fundamental interaction.

The ansatz for the metric is taken as

$$ds^2 = \beta^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{2}$$

where $(\eta_{\mu\nu}) = \mathrm{diag.}(-1, +1, +1, +1)$. The Einstein tensors become,

$$G_{\mu\nu} = g_{\mu\nu}\left[3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right)\right],$$
$$G_{55} = 6\left(\frac{\beta'}{\beta}\right)^2. \tag{3}$$

where prime denotes differentiation with respect to y. With the brane tension Λ_1 at B1 and the bulk cosmological constant Λ_b, the energy momentum tensors are

$$T_{MN} = -g_{MN}\Lambda_b - g_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(y) + 4\cdot 4!\left(\frac{4}{H^4}H_{MPQR}H_N{}^{PQR} + \frac{1}{2}g_{MN}\frac{1}{H^2}\right). \tag{4}$$

H_{MNPQ} has been considered before in connection with the cosmological constant problem [6] and compactification [7]. The specific form for $H^2 \equiv H_{MNPQ}H^{MNPQ}$ in Eq. (1) makes sense only if H^2 develops a vacuum expectation value at the order of the fundamental mass scale. Because of the gauge invariant four index H_{MNPQ}, four space-time is singled out from the five dimensions [7]. The four form field is denoted as G_4, or $H_{\mu\nu\rho\sigma}$,

$$G_4 = \sqrt{-g}\frac{\epsilon_{\mu\nu\rho\sigma}}{n(y)} \tag{5}$$

where $\mu, \cdots$ run over the Minkowski indices 0, 1, 2, and 3. In the 5D space, the three index antisymmetric tensor field is basically a scalar field a defined by $\partial_M a = (1/4!)\sqrt{-g}\epsilon_{MNPQR}H^{NPQR}$.

In this Letter, we show that there exists a solution for $\Lambda_b < 0$. The two relevant Einstein equations are the (55) and $(\mu\mu)$ components,

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - \frac{\beta^8}{A} \tag{6}$$
$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1\delta(y) - 3\frac{\beta^8}{A} \tag{7}$$

where A is a positive constant in view of Eq. (5). It is easy to check that the bulk equation is satisfied for any Λ_b, Λ_1, and A. This property is of the specialty of the H_{MNPQ} field. If we took H^2 (instead of $1/H^2$ term) in the Lagrangian, this statement will still hold but the resulting solutions do not lead to a self-tuning solution [8]. This interesting situation arises only for the β^8 ($1/H^2$ term in the action) or β^{-8} (H^2 term in the action) dependence of the A term in Eqs. (6) and (7), which is possible for the H_{MNPQ} field. Near B1(the $y = 0$ brane), the δ function must be generated by the second drivative of β. The Z_2 symmetry, $\beta(-y) = \beta(y)$, implies $(d/dy)\beta(y)|_{0^+} = -(d/dy)\beta(y)|_{0^-}$. Thus,

$$\frac{d^2}{dy^2}\beta(|y|) = \frac{d^2}{dy^2}\beta(|y|)\Big|_{y\neq 0} + 2\delta(y)\frac{d}{d|y|}\beta(|y|). \tag{8}$$

This δ-function condition at B1 leads to a boundary condition

$$\frac{\beta'}{\beta}\Big|_{y=0^+} \equiv -k_1, \tag{9}$$

where we define k's in terms of the bulk cosmological constant and the brane tension,

$$k \equiv \sqrt{-\frac{\Lambda_b}{6}}, \quad k_1 \equiv \frac{\Lambda_1}{6}. \tag{10}$$

It is sufficient if we find a solution for the bulk equation Eq. (6) with the boundary condition Eq. (9). We define a in terms of A,

$$a = \sqrt{\frac{1}{6A}}. \tag{11}$$

We note that the solution $\beta(y)$ should satisfy:

(i) the metric is well-behaved in the whole region of the bulk, and

(ii) the resulting 4D effective Planck mass is finite.

The solution of Eq. (6) consistent with the Z_2 symmetry is

$$\beta(|y|) = \left(\frac{k}{a}\right)^{1/4} [\cosh(4k|y| + c)]^{-1/4}, \tag{12}$$

where c is an integration constant to be determined by the boundary condition Eq. (9). This solution, consistent with (i), is possible for any value of the brane tension Λ_1. Note that c can take any sign. This solution gives a localized gravity consistent with the above condition (ii). The boundary condition (9) determines c in terms of Λ_b and Λ_1,

$$c = \tanh^{-1}\left(\frac{k_1}{k}\right) = \tanh^{-1}\left(\frac{\Lambda_1}{\sqrt{-6\Lambda_b}}\right). \tag{13}$$

The effective 4D Planck mass is finite

$$M_{P,\mathrm{eff}}^2 = 2M^3\left(\frac{k}{a}\right)^{1/2}\int_0^\infty dy\frac{1}{\sqrt{\cosh(4ky+c)}} = \frac{M^3}{\sqrt{2ka}}F\left[\alpha,\frac{1}{\sqrt{2}}\right]_0^\infty$$

$$= \frac{M^3}{\sqrt{2ka}}\int_{\sqrt{1-(\cosh(c))^{-1}}}^1 \frac{dx}{\sqrt{(1-x^2)(1-\frac{1}{2}x^2)}}. \tag{14}$$

Here $F(\alpha, r)$ is the elliptic integral of the first kind and

$$\alpha = \sin^{-1}\sqrt{(\cosh(4ky+c)-1)/(\cosh(4ky+c))}. \tag{15}$$

Note that the Planck mass is given in terms of the integration constant a, or the integration constant is expressed in terms of the fundamental mass M and the 4D Planck mass $M_{P,\mathrm{eff}}$,

$$a = \left(\frac{M^3}{M_{P,\mathrm{eff}}^2\sqrt{2k}}F\left[\alpha,\frac{1}{\sqrt{2}}\right]_0^\infty\right)^2. \tag{16}$$

To obtain the field equation for A_{MNP}, we note that the variation of of the Lagrangian (1) with respect to A^{NPQ} gives

$$\delta\mathcal{L} \supset -4\cdot 4!\,\partial^M\left(\sqrt{-g}\frac{H_{MNPQ}\delta A^{NPQ}}{H^4}\right) + 4\cdot 4!\left[\partial^M\left(\sqrt{-g}\frac{H_{MNPQ}}{H^4}\right)\right]\delta A^{NPQ}. \tag{17}$$

To cancel the first term of the above equation, we add a surface term in the action

$$S_{\mathrm{surface}} = \int d^4x dy\ 4\cdot 4!\ \partial^M\left(\sqrt{-g}\frac{H_{MNPQ}A^{NPQ}}{H^4}\right)$$

$$= \int d^4x dy\ 4\cdot 4!\left[\frac{\sqrt{-g}}{H^2} + A^{NPQ}\partial^M\left(\sqrt{-g}\frac{H_{MNPQ}}{H^4}\right)\right] \tag{18}$$

where the variation of derivative of A^{NPQ} vanishes at the boundary [9]. Then the field equation for A_{NPQ} is

$$\partial^M\frac{\sqrt{-g}H_{MNPQ}}{H^4} = 0, \tag{19}$$

which can be integrated to give

$$\frac{\sqrt{-g}H_{MNPQ}}{H^4} = \text{function of } y \text{ only}. \tag{20}$$

Due to our ansatz for the 4D homogenious space, H_{MNPQ} can have nonvanishing values only for $H_{\mu\nu\rho\sigma}$ as discussed in Eq. (5). Thus, A in Eqs. (6) and (7) and hence a in Eq. (11) is an integration constant. Field equations do not determine a, namely a is not dynamically determined. But a can take any value. Then for a given a, the Planck mass is given in terms of a as shown in Eq. (14). It is clear that this integration constant a itself does not participate in the self-tuning. On the other hand, the integration constant c participates in the self-tuning.

Suppose we are given with Λ_1 and Λ_b. Then we can always find a solution for $\Lambda_b < 0$ and for any value of Λ_1. Namely, there exists a flat space solution (2) with c given by Eq. (13). If we add some constant vacuum energy at B1, then Λ_1 is shifted to say Λ_1'. For this new set of Λ_1' and Λ_b, again we can find a solution, but with a different integration constant c' given with Λ_1' through Eq. (13). In other words, the dynamics of gravity and the antisymmetric tensor field adjust solutions a little bit, i.e. self-tune the above integration constant from c to c', to satisfy the field equations.

Even though we obtained a flat space solution for the 4D Minkowski space, it is worthwhile to check explicitly that the effective cosmological constant vanishes. From the action (1), the 4D gravity with vacuum energy is effectively described by

$$S = \int d^4x dy \sqrt{-\eta}\beta^4 \left[\frac{1}{2}\beta^{-2}\tilde{R}_4 - 4\left(\frac{\beta''}{\beta}\right) - 6\left(\frac{\beta'}{\beta}\right)^2 - \Lambda_b + \frac{2\cdot 4!}{H^2} - \Lambda_1 \delta(y) \right] + S_{\rm surface} \quad (21)$$

where the 4D metric is $\tilde{g}_{\mu\nu} = \beta^2 \eta_{\mu\nu}$, η is the determinant of $\eta_{\mu\nu}$, and $\tilde{R}_4$ is the 4D Ricci scalar. Then $-\Lambda_{\rm eff}$ is given by the y integral of Eq. (21) except the $\tilde{R}_4$ term,

$$\Lambda_{\rm eff} = \int_{-\infty}^{\infty} dy\ \beta^4 \left[4\left(\frac{\beta''}{\beta}\right) + 6\left(\frac{\beta'}{\beta}\right)^2 + \Lambda_b + \frac{\beta^8}{A} + \Lambda_1\delta(y) + \frac{2\beta^8}{A} \right]. \quad (22)$$

Using Eqs. (6) and (7), we can rewrite $\Lambda_{\rm eff}$ as

$$\Lambda_{\rm eff} = \Lambda_{\rm eff}^{(1)} + \Lambda_{\rm eff}^{(2)}, \quad (23)$$

where

$$\Lambda_{\rm eff}^{(1)} = -\int_{-\infty}^{\infty} dy \left(\frac{2}{3}\Lambda_b + \frac{1}{3}\Lambda_1\delta(y) \right)\beta^4, \quad \Lambda_{\rm eff}^{(2)} = -\frac{8}{3A}\int_0^{\infty} dy \beta^{12}. \quad (24)$$

Using the solution (12), and conditions (10), (11) and (13), we can show that

$$\begin{aligned}
\Lambda_{\rm eff}^{(1)} &= -2\frac{kk_1}{a}\frac{1}{\cosh(c)} + \left[2\frac{k^2}{a}\tan^{-1}\sinh(4ky+c)\right]_0^\infty \\
\Lambda_{\rm eff}^{(2)} &= 2\frac{k^2}{a}\frac{\sinh(c)}{\cosh^2(c)} - \left[2\frac{k^2}{a}\tan^{-1}\sinh(4ky+c)\right]_0^\infty ,
\end{aligned} \tag{25}$$

which leads to $\Lambda_{\rm eff}^{(1)} + \Lambda_{\rm eff}^{(2)} = 0$, in agreement with the flat 4D metric, Eq. (2).

We note that there have been attempts to self-tune the cosmological constant recently [10], but their solutions are not as simple as ours discussed in this paper or do need one fine-tuning.

In conclusion, we obtained a solution for self-tuning of the cosmological constant in the 5D theory with the Z_2 symmetry. For the self-tuning solution to exist, the bulk cosmological constant must be negative, $\Lambda_b < 0$, and we need a specific form for the gravitational interaction of the three index antisymmetric tensor field A_{MNP}.

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